

Name



WINCHESTER
COLLEGE

Winchester Election

Mathematics II

Wednesday 28th April 2021, 0900-1030

Time allowed: 1 hour 30 minutes (+5 minutes)

You have an additional 5 minutes to give you time to draw out diagrams, which may help with some of the questions.

Total marks: 100

Calculators are not allowed.

Write your answers on A4 paper. You may use a pencil for diagrams. You should show all your working so that credit may be given for partially correct answers.

Work carefully, and *do not be discouraged if you do not finish.*

Diagrams in the questions have not been drawn to scale.

Please start your working for each question on a new sheet of paper, and write your name at the top of each sheet of paper.

Please start Q1 on a new sheet of paper and write your name at the top.

1.	a) Evaluate $222 \div \frac{1}{5}$.	b) Evaluate $\sqrt{\sqrt{\frac{16}{81}}}$.	[1] [1]
	c) Write $\frac{1}{72} + \frac{3}{40}$ as a simple fraction.	d) Evaluate $\frac{0.14 \times 0.21}{0.00049}$.	[2] [2]
	e) Evaluate $(2\sqrt[3]{5})^3$.	f) $\frac{1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3}{1 + 2 + 3 + 4 + 5 + 6}$.	[2] [2]

Please start Q2 on a new sheet of paper and write your name at the top.

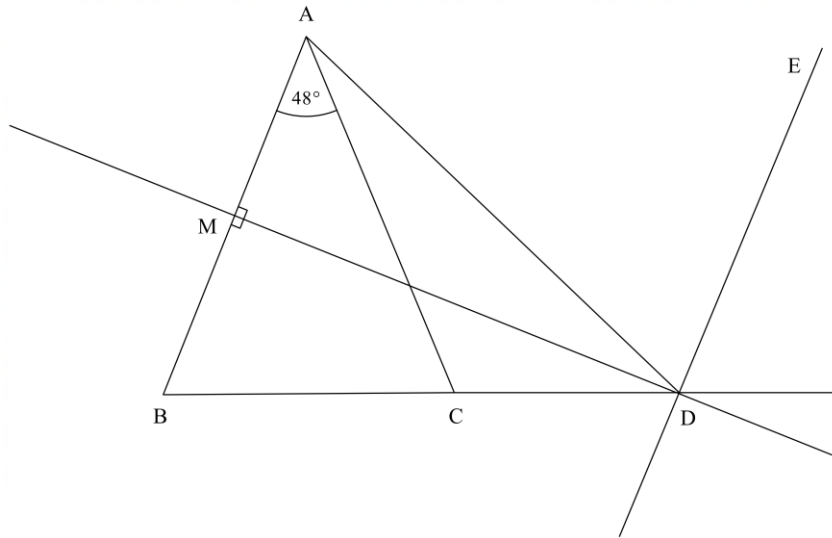
2.	$a = \frac{5}{2-b}$ and $b = \frac{5}{2-c}$.		
	a) What is a when b is -8 ?	b) What is b when a is $\frac{1}{3}$?	[2] [2]
	c) What is a when c is 12 ?	d) What is b when $a + c = 2$?	[2] [2]

Please start Q3 on a new sheet of paper and write your name at the top.

3.	a) Evaluate $(1+3+5+7+9+11)-(2+4+6+8+10+12).$	b) Evaluate $\left(\frac{(-18)^{11}}{9^{11}} + \frac{(-12)^{11}}{(-6)^{11}} + 1\right)\left(\frac{(-14)^7}{(-7)^7} - \frac{(-16)^7}{8^7} + 1\right).$	[2] [2]
	c) Expand and simplify $\left(x + \frac{1}{x}\right)\left(x^2 - 1 + \frac{1}{x^2}\right).$	d) Given that $x + \frac{1}{x} = 5$ find $x^3 + \frac{1}{x^3}.$	[2] [3]

Please start Q4 on a new sheet of paper and write your name at the top.

4. The triangle ABC is isosceles ($AB = AC$). DM is a perpendicular bisector of AB (M is the midpoint of AB). DE is parallel to AB.



Find the size of the following angles.

a) BDM

[2]

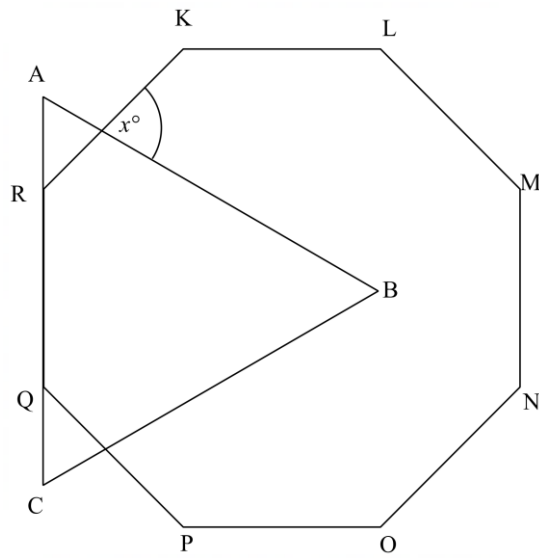
b) ADE

[3]

Please start Q5 on a new sheet of paper and write your name at the top.

5. ABC is an equilateral triangle. KLMNOPQR is a regular octagon.
Find the angle x .

[3]

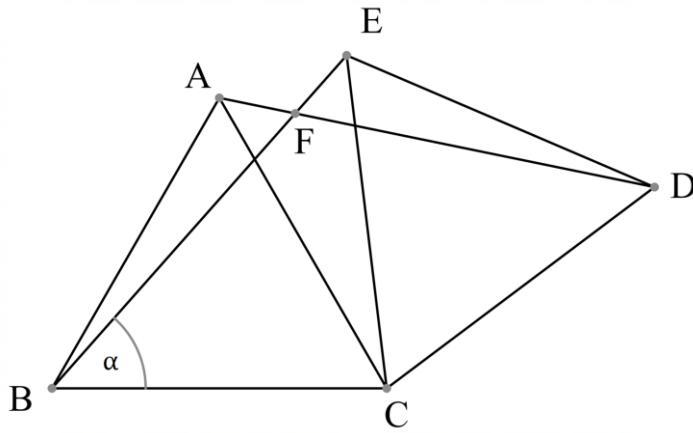


Please start Q6 on a new sheet of paper and write your name at the top.

6.	a) What is the remainder when $2 \times 3 \times 5 \times 7 \times 11$ is divided by 4?	[1]
	b) What is the remainder when $3 \times 13 \times 23 \times 43$ is divided by 6?	[1]
	c) What is the remainder when $2^2 \times 23^2 \times 223^2$ is divided by 8?	[2]
	d) $2 \times 3 \times 5 \times 7 \times 11 \times 13 \times 17 \times 19 \times 23$ leaves the same remainder when divided by 8 as it does when divided by 18. What is the remainder?	[3]

Please start Q7 on a new sheet of paper and write your name at the top.

7. In the diagram below, ABC and CDE are equilateral triangles and the straight lines AD and BE meet at F . Angle EBC is called α .



Find the following angles, leaving your answers in terms α where appropriate.

a) EBA

[1]

b) BAF

[2]

c) EFA

[1]

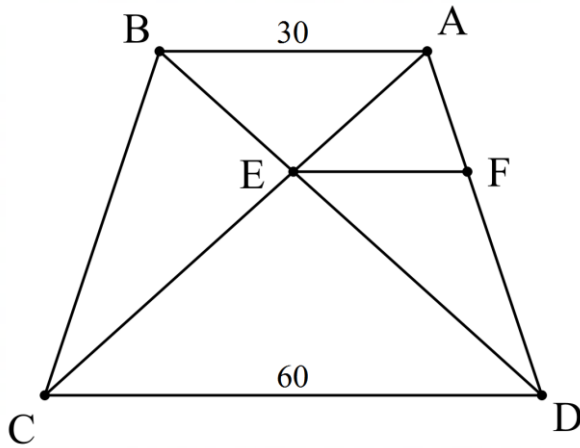
Please start Q8 on a new sheet of paper and write your name at the top.

8. ABCD is a trapezium whose diagonals meet at E. The length of AB is 30, and the length of CD is 60. The lines AB, CD and EF are all parallel.

a) Find the length of EF.

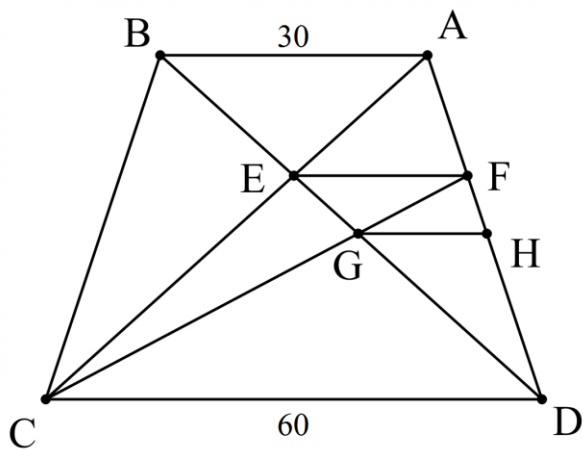
(Hint: you may wish to consider the pair of triangles ABE and CDE and the pair AEF and ACD)

[2]



b) When the line CF is added to the diagram, it meets BD at G. The line GH is parallel to EF. Find the length of GH.

[2]

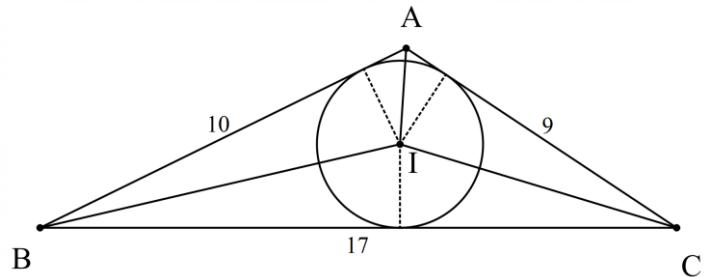


Please start Q9 on a new sheet of paper and write your name at the top.

9. The diagram shows triangle ABC whose sides have lengths 9, 10 and 17. The circle with centre I touches all three sides of the triangle.

- a) Given that the area of the triangle is 36, find the radius of the circle.
(Hint: you may wish to consider the triangles ABI, BCI and CAI.)

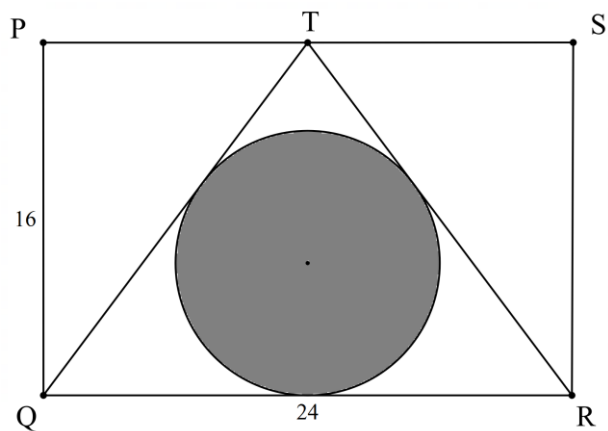
[3]



- b) The diagram shows a rectangle PQRS whose sides have lengths 16 and 24. T is the midpoint of PS. The shaded circle touches all three sides of triangle QRT and has area $k\pi$.

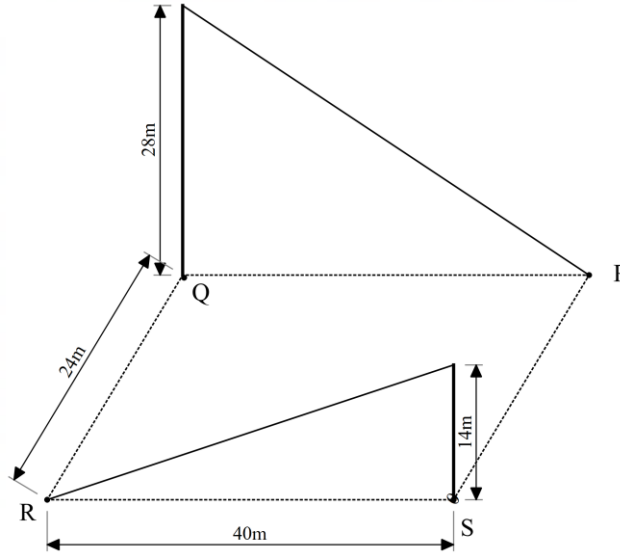
Find the value of k .

[5]



Please start Q10 on a new sheet of paper and write your name at the top.

10. PQRS is a rectangular field with $PQ=RS=40\text{m}$ and $QR=PS=24\text{m}$. There is a 28m vertical pole at Q, the top of which is joined to P by a straight wire. There is a 14m vertical pole at S, the top of which is joined to R by a straight wire. Ariadne the spider starts at R and crawls half way up the wire before stopping for a rest. While Ariadne is resting, a second spider called Charlotte starts crawling up the wire from P.



a) How far apart are the spiders when they are the same height above the ground? [3]

b) How far apart are the spiders when they are both half way up their wires? [3]

When Charlotte is 80% of the way up her wire, she attaches a thread to the wire and descends vertically 740cm.

c) How far apart are the spiders now? [4]

Please start Q11 on a new sheet of paper and write your name at the top.

11. The sequence of Fibonacci numbers begins with two 1s and each subsequent number is the sum of the previous two. The sequence begins:

1, 1, 2, 3, 5, 8, 13, ...

a) Suppose that x and y are two consecutive Fibonacci numbers. Fill in the five blanks below to complete the list of seven Fibonacci numbers, leaving your answers in terms of x and y .

[2]

____ , x , y , ____ , ____ , ____ , ____

b) Is it possible for both x and y to be even? Explain your reasoning.

[2]

c) How many of the first 600 Fibonacci numbers are even?

[2]

d) Suppose that x is divisible by 5 and y is not. Write down the next Fibonacci number after y which is a multiple of 5, leaving your answer in terms of x and y .

[2]

e) How many of the first 600 Fibonacci numbers are divisible by 30?

[2]

f) The k^{th} Fibonacci number is a multiple of 390. What is the smallest possible value of k ?

[2]

Please start Q12 on a new sheet of paper and write your name at the top.

12. An *anagram* is an arrangement of the letter of a particular word. For this question anagrams do not need to be real English words. For example 'TOP' is an anagram of 'POT', but so are 'OPT', 'TPO', 'PTO' and 'OTP'.

a) List all the anagrams of the word 'FIG' in alphabetical order. (*You should include the original word in your list.*)

[1]

b) How many anagrams of the word 'PLUM' are there?

[1]

c) All anagrams of the word 'PEACH' are listed in alphabetical order. 'ACEPH' is number 2 in the list. What number in the list is 'CAEHP'?

[2]

d) All anagrams of the word 'LEMONS' are listed in alphabetical order. What is the 243rd anagram in the list?

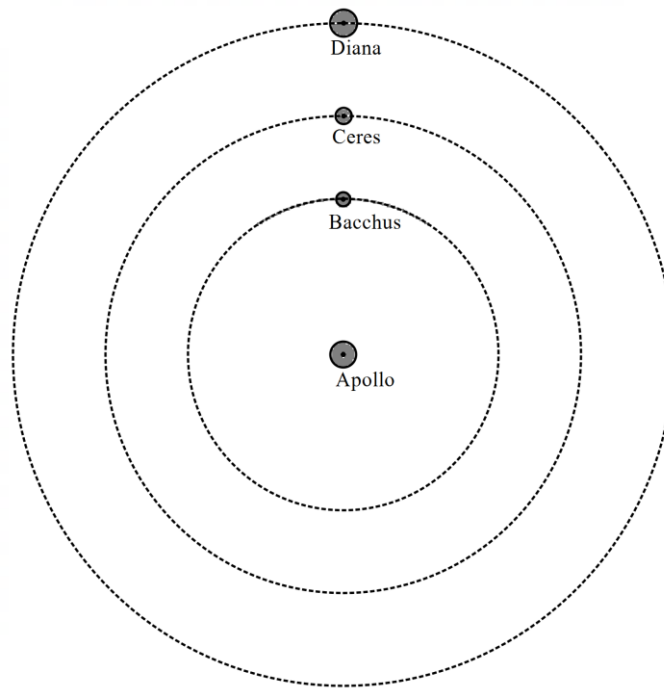
[3]

e) All anagrams of 'ORANGE' are listed in alphabetical order. What number is 'ORANGE' in the list?

[3]

Please start Q13 on a new sheet of paper and write your name at the top.

13. Bacchus (B), Ceres (C) and Diana (D) are three planets that perform circular orbits around the star, Apollo (A), at constant speeds.



Initially all four bodies, A, B, C and D are in a line, as shown in the diagram.

C always moves clockwise around A performing two revolutions every Earth year.

- a) B moves clockwise around A performing one revolution every Earth year. How long until A, B and C are once again in a line (in that order)?

[2]

- b) B moves clockwise around A performing **five** revolutions every Earth year. How long until A, B and C are once again in a line (in that order)?

[2]

c) B moves **anticlockwise** around A performing five revolutions every Earth year. How long until A, B and C are once again in a line (in that order)?

[2]

d) B moves **clockwise** around A performing five revolutions every Earth year. D moves clockwise around A performing one revolution every three Earth years. How long until A, B, C **and D** are once again in a line (in that order)?

[4]